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GRAPH CHARACTERIZATIONS IN PRIME SQUARE DOMINATION

S. Senthilraj
Department of General Requirements Subjects (Mathematics Section),
Sur-College of Applied Sciences, Ministry of Higher Education,
Sur, Sultanate of Oman.Oman

ABSTRACT
This paper deals with the study of different edge domination numbers of a prime square domination graphs and a prime dominating graph not necessarily a prime square dominating graph, since, every prime square domination graph is a prime domination graph. In addition, some of the classified prime square domination graphs are selected for this study, their edge domination numbers are resolved and to discuss some of their domination parameters. The proposed theorems established some domination numbers and complementary tree domination number of the Prime Square Dominating Graphs.

KEYWORDS- Dominating set, domination graph, prime dominating graph, prime square dominating graph, edge dominating set, edge domination number, independent edge domination number, complementary tree dominating set, complementary tree domination number, minimal prime square dominating graphs.

INTRODUCTION

Graph theory is the most beautiful branches of recent mathematics with an enormous range of research scope. Most of the graph theory problems are modeled from a real life problems, it can be represented graphically with a set of points called as vertices joined together with lines called as edges. For example, in network problems, network places can be represented by vertices and network links can be represented by edges, so in the real life situations it can be understood in a better appearance. For more than three decade, the growth of graph theory has lot of applications to describe algebraic and combinatorial problems. Graph Theory is the subject, which can be very much useful to understand the real life situations in a better manner.

The graph $G = (V, E)$ represents undirected, connected and simple graphs.The important areas of research in graph theory are domination and the graph labeling. The domination problem in a graph is to find a minimum sized vertex set D such that no vertex set is in D is adjacent to at least one vertex set in D. The domination number of the graph G, represented by $\gamma(G)$, whichthe minimum cardinality of a dominating set isof G. Dominating sets play a vital role in algorithms and combinatorial. Complementary tree dominating set problem is the problem of finding complementary tree dominating set of specific size for the given graph. In a graph G with vertex set $V(G)$, a dominating set S is a subset of V and edge set $E(G)$ is a complementary tree dominating set of G if the induced sub graph $< V - S >$ is a tree. Complementary tree domination number is the minimum cardinality of a complementary tree dominating set G,which is denoted by$\gamma_{ctd}(G)$. To obtain some of the results pertaining to the bounds of complementary tree domination number. Some complementary tree domination number of specified class of interval graphs and circular arc graphs obtained in [8].

A subset P of E is called an edge dominating set of G, if every edge not in P is adjacent to some edge in P. The edge domination number $\gamma'(G)$ of G is the minimum cardinality taken over all edge dominating sets of G. An edge dominating set Pis called an independent edge dominating set if no two edges in P are adjacent. The independent edge domination number $\gamma_i'(G)$ of G is the minimum cardinality taken over all independent edge dominating sets of G. The edge independent number $\beta_1(G)$is defined to be the number of edges in a maximum independent set of edges. Labeling of a graph plays a vital role in graph theory. A labeling of a graph G is an assignment of distinct positive integers to its vertices. A graph is a prime square dominating graph, if the vertices of graph G are labeled with positive integers, such that the vertex labeled with composite number c is adjacent to the vertex named with prime number p if and only if p^2/c. In this paper, the study of some characterization in domination and Prime Square dominating graphs discussed.

DEFINITIONS

- A linear graph $G = (V, E)$ consists of a set of vertices $V = \{v_1, v_2, \dots,\}$, and another set edges $E = \{e_1, e_2, \dots\}$, such that each edge e_k identified with an unordered pair (v_i, v_j) of vertices. v_i. If(V, E) is a finite sets, then the graph is called a finite graph.
- A walk is defined as a finite consecutive sequence of vertices and edges, edge repetitions not allowed then beginning and ending with vertices, such that each edge is incident with the vertices preceding and following it.
- A path is a walk, if the vertices $v_0, v_1, \dots, v_k$ of the walk $v_0 e_1 v_1 e_2 v_2$ ... e_k are distinct.
- If there is at least one path between every pair of vertices in G, the G is called connected graph, .if it is not connected then it is said to be disconnected.
- A graph $G$is called a bipartite graph, if the vertex set can be partitioned in to two sets such that every edge of G having one end in one vertex set and another in other vertex set.
- In a graph G, a subset S of $V(G)$ is said to be a dominating set if every vertex not in S has a neighbor in S. The domination number which denote by γ (G) as $min\{|S|/\ S\ is\ a\ dominating\ set\ in\ G\}$.
- A graph $G(V, E)$ be a prime dominating graph, if for any two vertices x, y in the vertex set V of graph G with $f(x) > f(y)$, are adjacent if $f(y)$ divides $f(x)$ and $f(y)$ is prime number.
- A Graph is Prime Square dominating graph, if the vertices of graph G is labeled with positive integers and the vertices labeled with composite number $n$is adjacent to the vertices labeled with prime numbers $p$if and only if $n$is divided by p^2.
- Every prime square dominating is a finite and simple graph and also it is a prime dominating but converse is not true.

PRIME SQUARE DOMINATION THEOREMS

Theorem 1: Any graph containing an odd cycle is not prime square dominating graph.

Proof: Let G be a graph of odd cycle vertices, if G has Prime Square dominating then the vertices labeled by prime number and composite number alternatively like$v_1,\ v_2,\ v_3, \dots\dots v_{2n-1}$. So v_1 and v_{2n-1} were connected and both have same kind of label like both are prime number or both are composite number. This means that no two prime numbers or composite numbers can be connected together. So in a prime square dominating labeling no graph has containing odd cycles. Thus, any graph containing an odd cycle is not prime square dominating graph.

Theorem 2: A graph is a prime square dominating graphif and only if it is a bipartite graph.

Proof: Let G be a bipartite graph. The vertex set V of graph Gcan be partitioned into V_1 and V_2 sets. Therefore, every edges of graph G having one end in vertex v_1 and another end in v_2. Let $|V_1| = m$ and$|V_2| = n$. Suppose all vertices of $V_1$are labeled prime and V_2 are labeled composite depends on$V$, so G is a prime square dominating graph. Conversely, if G is a prime square then as per the above theorem 1, it is not containing odd cycle, so it is bipartite.

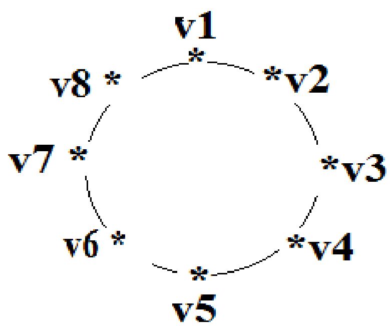
Theorem 3: Every cycle graph with even number of vertices is a prime square dominating graph.

Proof: Let $V = \{v_1,\ v_2,\ v_3, \dots\dots v_{2n}\}$ with C_{2n} cycle in the graph, $v_1,\ v_3,\ v_5, \dots\dots v_{2n-1}$ are the prime numbers $P_2,\ P_3, P_5, \dots\dots P_{2n-1}$ respectively and the vertices $v_2,\ v_4,\ v_6, \dots\dots v_{2n}$ with composite number

C_2, C_4, C_6 ,…..C_{2n} respectively. So , P_2^2 divide $C_{2n} and C_2$, P_3^2 divide $C_2 and C_4$, P_5^2 divide $C_4 and C_8$ . . . and P_{2n-1}^2 divide $C_{2n-1} and C_{2n}$.

Therefore, it is clear that the graph C_{2n} labeled with composite number P is P_c^2. Therefore, the vertices of every cycle graph with even number of vertices can be labeled in such a way that the graph becomes prime square dominating graph. In other ways, a cycle graph with odd number of vertices is not a prime square dominating graph.

Let a cycle graph C_8 with partition graph shown in the following figure $Fig.(3.1)$ and $Fig.(3.2)$.



$Fig. 1$ $Cyclegraph C_8$

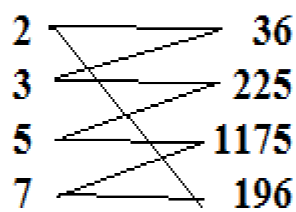


Fig. 2 PrimesquaredominatinggraphC_8

Put v_1, v_3 , v_5, v_7 be the prime numbers 2, 3, 5 *and* 7 , then v_2, v_4, v_6 *and* v_8 with composite number36, 225, 1175 *and* 196. It is very much clear that the prime number 2 is adjacent to 36 and 196 and $2^2$is divided by both 36 and 196. Similarly, 3 is adjacent to 36 and 225 and 3^2 is divided by 36 and 225, 5 is adjacent to 225 and 1175 and 5^2 is divided by both 225 and 1175. Finally, 7 are adjacent to 196 and 1175 and 7^2 are divided by both 196 and 1175. Hence, C_8 is a prime square dominating graph and it implies every cycle graph with even number vertices is a prime square dominating graph.

Theorem 4: Every tree is a prime square dominating graph.

Proof: Let G be a graph and T be a tree of height with h. Let the height of the tree be even and odd.

Case (i): Ifh may be even. Partition of the vertex set V into two subsetsV_1 *and* V_2. Such that V_1= set of all vertices at level 0, 2, 4, … , h and V_2= set of all vertices at level 1, 3, 5, …, h-1.

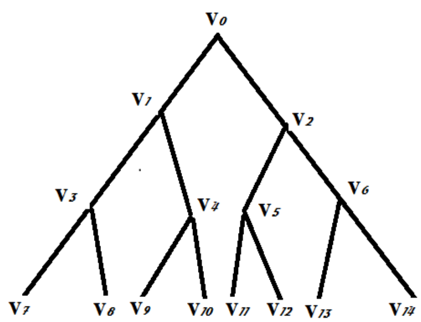
Case (ii):Ifh may be odd. Partition of the vertex set V into two subsets. Such that V_1= set of all vertices at level 0, 2, 4, … … , h-1 and V_2= set of all vertices at level 1, 3, 5, ……, h.

The above two cases, the vertex set V of the graph is partitioned into two sets V_1 *and* V_2. So that every edge of the graph has one end vertex in V_1 and another end vertex in V_2 . If we labeled all the vertices in V_1 *with* prime numbers and all the vertices with composite numbers depending upon the labeling of the vertices in

V_1 , Such that the vertex labeled with composite number c,which is adjacent to the vertex labeled with prime number p, if and only if p^2/c.By the definition of prime square dominating graph, the tree T becomes a prime square dominating graph. It follows that, every tree is a prime square dominating graph.

Let the tree be the partition of the vertex set into two subsets V_1 *and* V_2. Such that, V_1 be the set of all vertices at level 0 and 2, then V_2be the set of all vertices at level 1and 3. That means, $V_1 = \{v_0,\ v_3, v_4,\ v_5,\ v_6\}$ and $V_2 = \{v_1,\ v_2, v_7,\ v_8,\ v_9,\ v_{10}, v_{11},\ v_{12},\ v_{13}, v_{14}\}$. we can label the vertices of the tree T with prime numbers 11, 2, 3, 5 and 7 as $v_0,\ v_3, v_4,\ v_5$ *and* $v_6$respectively, then the vertices with composite numbers 4356, 148225, 4, 6, 9, 18, 175, 49 and 245 respectively. Clearly, the vertex labeled with composite number 4356 is adjacent to vertices labeled with prime numbers 11, 2 and 3; vertex labeled with composite number 148225 is adjacent to vertices labeled with prime numbers 11, 5 and 7. Similarly, 4 and 6 are adjacent to the vertex labeled with prime number 2; 9 and 18 are adjacent to the vertex labeled with prime number 3; 25 and 175 are adjacent to the vertex labeled with prime number 5; finally, 49 and 98 are adjacent to the vertex labeled with prime number 7. From the definition of the prime square dominating graph, it follows that T is a prime square dominating graph.

Therefore, labeling of every Tree can do in such a way that the tree becomes a prime square dominating graph. The following figures [$Fig.(3.3)$ *and* $Fig.(3.4)$] shows the corresponding prime squares dominating tree and prime square dominating graph.



$Fig.(3)$

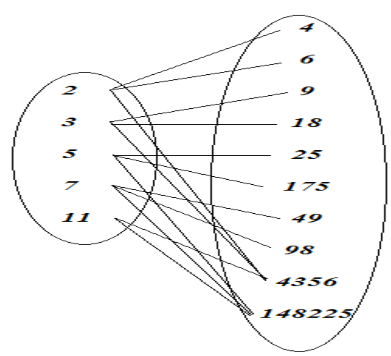


Fig. (4)

Theorem 5: In a vertex set $V = \{v_1,\ v_2,\ v_3, \ldots. v_n\}$, $n \geq 2$ be a prime square dominating graph G , such that a vertex v_i, $1 \leq i \leq n$ is labeled with a prime number $p_i$and the remaining all vertices are labeled with composite numbers $V = c_2,\ c_3, \ldots.. c_n$, so that p_1^2divides $c_k$for $k = 2, 3, 4, \ldots., n$. Then the edge domination number of the prime square dominating graph G is one.

Proof: Consider, G be a prime square dominating graph with the vertex set $V = \{v_1,\ v_2,\ v_3, \ldots.. v_n\}$, be labeled with composite numbers and prime numbers satisfying the conditions mentioned in the theorem. Say, $\{p_1,\ c_k\}$, is an edge of the graph for $k = 2, 3, 4, \ldots., n$. Since p_1^2dividesc_k, as there is none of the other vertex labeled with prime except p_1, $c_k$cannot be adjacent to any other vertex, except p_1, for $k = 2, 3, 4, \ldots., n$. Therefore, the distinct edges of the graph are $\{p_1, c_2\}, \{p_1, c_3\}, \ldots\{p_1, c_n\}$ and every edge is dominates every other edge of the graph. The set of every edge is an edge dominating set. Edge domination number of a graph is always more than or equal to one. Hence, edge domination number of the graph is one.

Let the prime square dominating graph G with vertex set V = {2, 4, 12, 20, 28}. The graph satisfies all the conditions stated in Theorem 4 for $p_1 = 2$, $c_2 = 4, c_3 = 12, c_4 = 20, c_5 = 28$, that means, minimum edge dominating sets are $\{p_1, c_k\}$, for $k = 2, 3, 4, 5$. then the edge domination number of the graph is one. Prime square dominating graph will be shown in the fi$g$. (3.5)

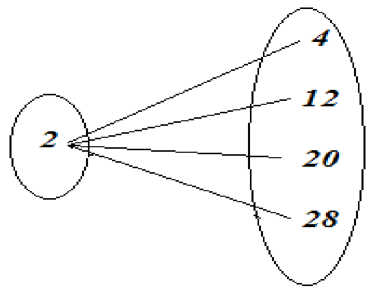


Fig. 5.

Theorem 6: If v_i and, v_j , where, $1 \leq i, j \leq n$ and $i \neq j$ of the vertex set $V = \{v_1, v_2, v_3, \ldots.. v_n\}, n \geq 4$ of a prime square dominating graph. The graph G are labeled with a prime number p_i and composite number c_j respectively and the remaining vertices are labeled with both composite and prime numbers so that p_i^2divides all the composite numbers and all the squares of primes divide only c_j , then the edge domination number of the prime square dominating graph G is one.

Proof: Let the vertex set $V = \{v_1, v_2, v_3, \ldots.. v_n\}$, of a prime square dominating graph G labeled with composite numbers and prime numbers satisfy the conditions mentioned in the theorem.

By, using the hypothesis, except p_i, the squares of all the other primes divide only one composite number c_j, by using the hypothesis.Therefore, all the vertices labeled with primes, except p_i, and are adjacent only toc_j . Moreover, all the composite numbers, except c_j, are divisible only by p_i^2. Vertex labeled with $p_i$is an adjacent to all the other vertices labeled with composite numbers and which is adjacent to all the other vertices labeled with prime numbers. It follows that, all the edges are adjacent to the edge $\{p_i, c_j\}$. The edge set consisting of single edge $\{p_i, c_j\}$ is an edge dominating set and further more it is a minimum edge dominating set. Hence, the edge domination number of the graph is one.

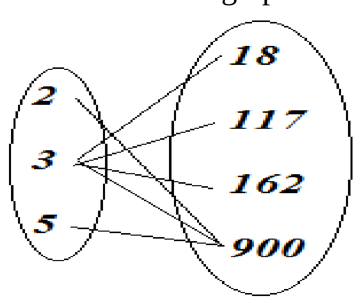


Fig. 6

Let G be the prime square dominating graph with vertex set$V = \{2, 3, 5, 18, 117, 162, 900\}$. Prime square dominating graph in this case will be as shown in the$Fig.$ 3.6. Here, 2^2 divide only 900, 3^2divides $18, 117, 162$ *and* 900. In addition, 5^2 divide only 900. The vertices $p_i = 3$ and $c_j = 900$ satisfy all the conditions mentioned in the Theorem 3.6. The edge $\{3, 900\}$ is adjacent to all the remaining edges. Hence, the set consisting of the single edge $\{3, 900\}$ is a minimum edge dominating set. Hence, the edge domination number of the graph is one.

CONCLUSION

Initially we noticed that the prime square domination graph is not a prime domination graph. We derived that if a graph contains a cycle of an odd, then it is not a prime square domination graph, also we noticed that if graph is bipartite then it is a prime square dominating graph. Then, solved some characterization of prime dominating graphs has been focused in this paper. Finally, some results regarding the edge domination in graphs and headed out into exploring the edge domination number of some prime square dominating graphs discussed. In future studies plan to put the ideas of a minimal prime square dominating graphs with chromatic numbers and relations between other domination parameters of a graph and prim square dominating graphs.

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